Dos & Don'ts

of EL Instruction

ENGLISH LEARNERS SUCCESS FORUM

GRADES PK–2

SUBJECT	AREA OF FOCUS		GUIDELINE		SPECIFICATION			
MATH	I	II	3	4	3A	3B	4A	4B

Developing Mathematical Discourse Through a Community of Young Bilingual Learners Who Talk and Do Mathematics

What we know about the topic

Young learners' capacity to engage in a wide range of mathematics word problems is often underestimated (Carpenter et al., 1999). When teachers create opportunities to engage in challenging mathematics word problems, young learners and emergent bilinguals can solve a broad range of problem types (e.g., Join Result Unknown, Separate Result Unknown, Join Change Unknown, Separate Change Unknown, Multiplication, Division, and Multi-Step Problems) (See Celedón-Pattichis & Turner, 2012; Turner & Celedón-Pattichis, 2011; Turner, Celedón-Pattichis, Marshall, and Tennison, 2009).

Problem Type	A mathematical story
Join *result unknown*	Ms. Mejia went to the market to buy apples. She picked up 4 apples and put them in the shopping cart. Her son, Nico, put another 4 apples in the shopping cart. How many apples did they have in the shopping cart?
Join *change unknown*	Marina goes to the store to buy a bag of candy. The bag of candy cost 5 dollars, but Marina only has 3 dollars. How many more dollars does Marina need to buy the bag of candy?
Multiplication	You and your two cousins are playing outside in the park and you find some nickels on the ground. You each picked up 3 nickels and you put them all together. You counted the nickels with your cousin. How many nickels did you and your two cousins find?
Separate *result unknown*	Albert has 7 baseballs and he gave 3 baseballs to Carlos. How many baseballs does Albert have left?

Featured Authors

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Division *partitive*	Ms. Chan has 15 red envelopes and she wants to give them to Mei Mei, Evelyn, and Paul. She wants each of them to get the same number of red envelopes. How many should she give to each person?

A mathematics discourse community of students who talk mathematics develops language and content simultaneously by (Moschkovich, 2012; Musanti & Celedón-Pattichis, 2013; Willey, 2013):

- engaging students in problem solving through storytelling and informal conversations;
- positioning students as problem solvers;
- providing multiple entry points to represent solutions using multimodal representations (e.g., drawings, symbols, writing); and
- accessing the home language so that students can express their mathematical thinking.

How might teachers begin this work? We offer one process teachers might use to engage students in solving story problems, while creating a community that talks mathematics.

Step 1: Provide Time to Play with Manipulatives and Represent Quantities

To support students in developing language to express themselves, provide time for students to explore different ways to represent quantities. Select students to share different solutions to represent, for example, the number 3. Strategically select students to present their problem-solving strategy, from simple to more sophisticated. This positions them as leaders and acknowledges the value of their contributions (Chval et al., 2021).

Step 2: Develop One-to-One Correspondence by Introducing Different Types of Story Problems

To support students in communicating mathematically, introduce addition and subtraction type problems that involve an action (i.e., *You have 4 strawberries. Your mom gave you one more. How many do you have total?*). The goal is to establish one-to-one correspondence as students learn to count different objects. In the beginning, the teacher will need to guide this process by posing questions such as: *How many strawberries do you have? How many did your mom give you?* Reminding students of the quantities in the problem and having students touch and count the objects will be important in learning one-to-one correspondence. Open-ended questions, such as *Show me or Why did you represent the number in this way?*, prompt students to clarify and expand their mathematical thinking. These steps support students to develop early number sense (Carpenter et al., 1999).

Step 3: Use Multimodal Representations to Solve Word Problems

Ask students to use drawings, tally marks, symbols, and equations to represent quantities and the solution to a word problem. Allowing students to choose how they

It is recommended that these practices be part of a comprehensive approach to EL instruction and not in isolation as laid out in our Guidelines for Improving Math Materials for English Learners.

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want to represent the solution gives them ownership of the strategy they use to solve the story problem (Celedón-Pattichis & Musanti, 2014). Asking students to explain their solutions and to discuss which strategies are more efficient than others builds the language skills needed for problem solving.

Step 4: Support Students to Move From Problem Solvers to Problem Posers

Figure 1 illustrates how students can be supported to shift from concrete to abstract mathematical concepts. This process includes composing story problems and expressing solutions by direct modeling (e.g., using manipulatives) and orally expressing their solutions to develop cardinality of numbers (e.g., knowing 5 represents five fingers on their hand) and written formats in their journal.

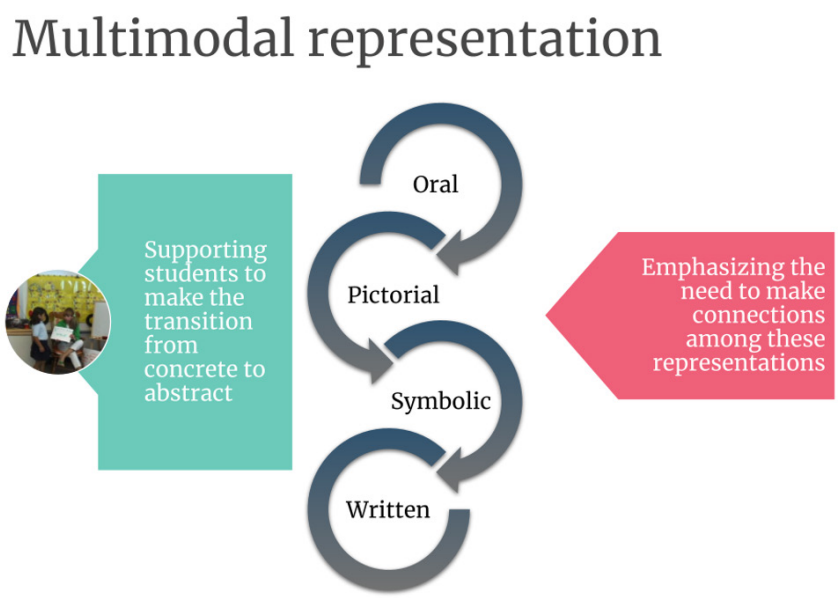

Figure 1: Multimodal Representations to Support Mathematics Learning (from Musanti & Celedón-Pattichis, 2013, p. 52)

Based on the research and ELSF guidelines, we suggest the following:	
DO draw from the students' language and culture to set up a familiar context and to make connections to mathematical word problems.	DON'T use unfamiliar language or contexts when introducing new concepts.
DO provide tools that students can use to represent their mathematical thinking (i.e., unifix cubes, counting bears, base-10 blocks).	DON'T stop providing tools for all students, even when they no longer ask to use them.

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DO provide a broad range of problem types for students to solve (i.e., addition, subtraction, multiplication, division, multi-step).	DON'T just give students a worksheet. It's important for students to create their own story problems.
DO develop oracy and literacy by having students retell the story in a word problem and check students' understanding of the quantities involved. Provide student support in the moment to use and learn new language to express their thinking.	DON'T underestimate a student's math story problem no matter how simple it may be. For example: *I had one chocolate and my mom gave me one chocolate.* Even if the student is struggling to express their thinking orally or in writing, don't ignore the student's potential to solve and retell the story in the math problem.
DO position students as competent problem solvers who have important mathematics contributions to make by calling on students to share their solution and by prompting active listening (i.e., *Listen to Juan's strategy and say what you liked about his solution. Ask a question about his strategy.*)	DON'T force students to speak in front of the class if they are not ready. Instead, create a safe environment where all students listen respectfully and build on each other's solutions.
DO ask close-ended questions to support or clarify students' mathematical thinking (e.g., "How many puppies did she have"? and open-ended questions to *extend* their reasoning (e.g., Show me, "why?").	DON'T use questions that elicit only memorized responses. Give students time to answer, and support them to think and reflect.
DO encourage students to write in journals, invent spelling, and play with language as needed to create story problems and pose these for other students to solve. Support students to bridge their own language to mathematical concepts and vocabulary.	DON'T just correct grammar or spelling without building student understanding of meaning or patterns in language.
DO prompt students to represent their solutions using multimodal representations (e.g., oral, pictorial, symbolic, written).	DON'T limit the student to one strategy or solution. Don't give the answer without letting students answer independently.

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Useful Steps to Create a Mathematics Discourse Community

Here is the example of one bilingual kindergarten classroom where the teacher, Karla, utilizes the four steps delineated above to set up a community that talks mathematics. In the fall semester, Karla introduces problem solving by having students work with manipulatives including unifix cubes and small counting bears. Karla uses realia in the story problems so that students can see different ways to represent the solutions. For example, "I have 5 friends and I give two lollipops to each friend. How many lollipops are there altogether?"

Karla introduces journals in February to work from concrete to more abstract concepts. She begins with addition and subtraction type problems because there is an action involved (i.e., I have, they gave me, I lost.). Students can directly model the solution with manipulatives. Karla exposes students to a broader range of problem types over time.

Although one student, Alexandra (pseudonym), did not refer to lollipops in her written problem, she constructed a story in which she had 5 lollipops and her mom gave her 4 more (see Figure 2). She represented these quantities with the drawing of the lollipops, the equation illustrating 5 + 4 = 9, and writing these quantities in a story. At the end of the spring semester, Alexandra used double digit numbers and understood the concept of 0. We see these results when students have opportunities to solve and pose story problems orally and in writing, and use different ways to express their thinking.

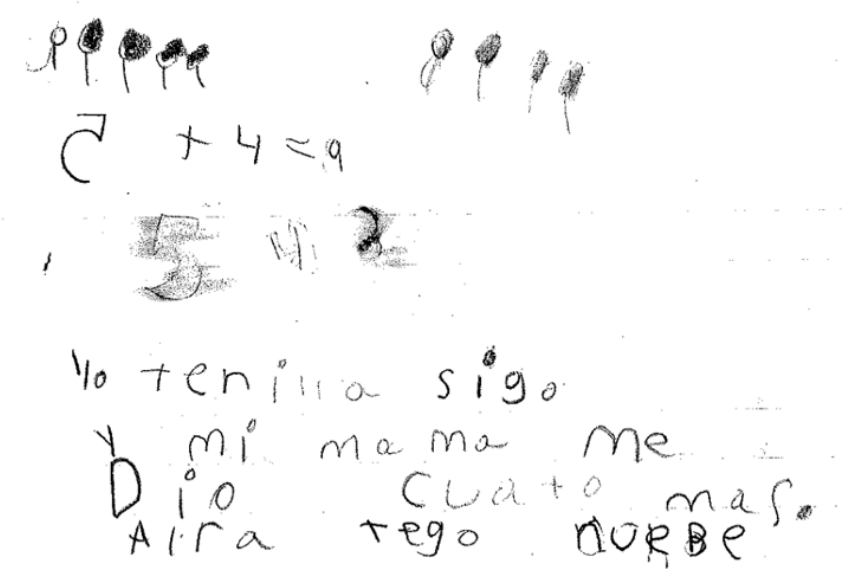

Figure 2: Alexandra's Journal Representing an Addition Problem.

This video clip (https://youtu.be/2gLrtI4TiIY) illustrates how a teacher supports students in developing multiplication concepts while developing mathematics discourse. The students had already established one-to-one correspondence and understood cardinality, and thus were able to use more sophisticated strategies such as counting by fives to solve multiplication type problems.

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